Middle Years Programme

Mathematics guide

For use from September 2014/January 2015

International Baccalaureate®
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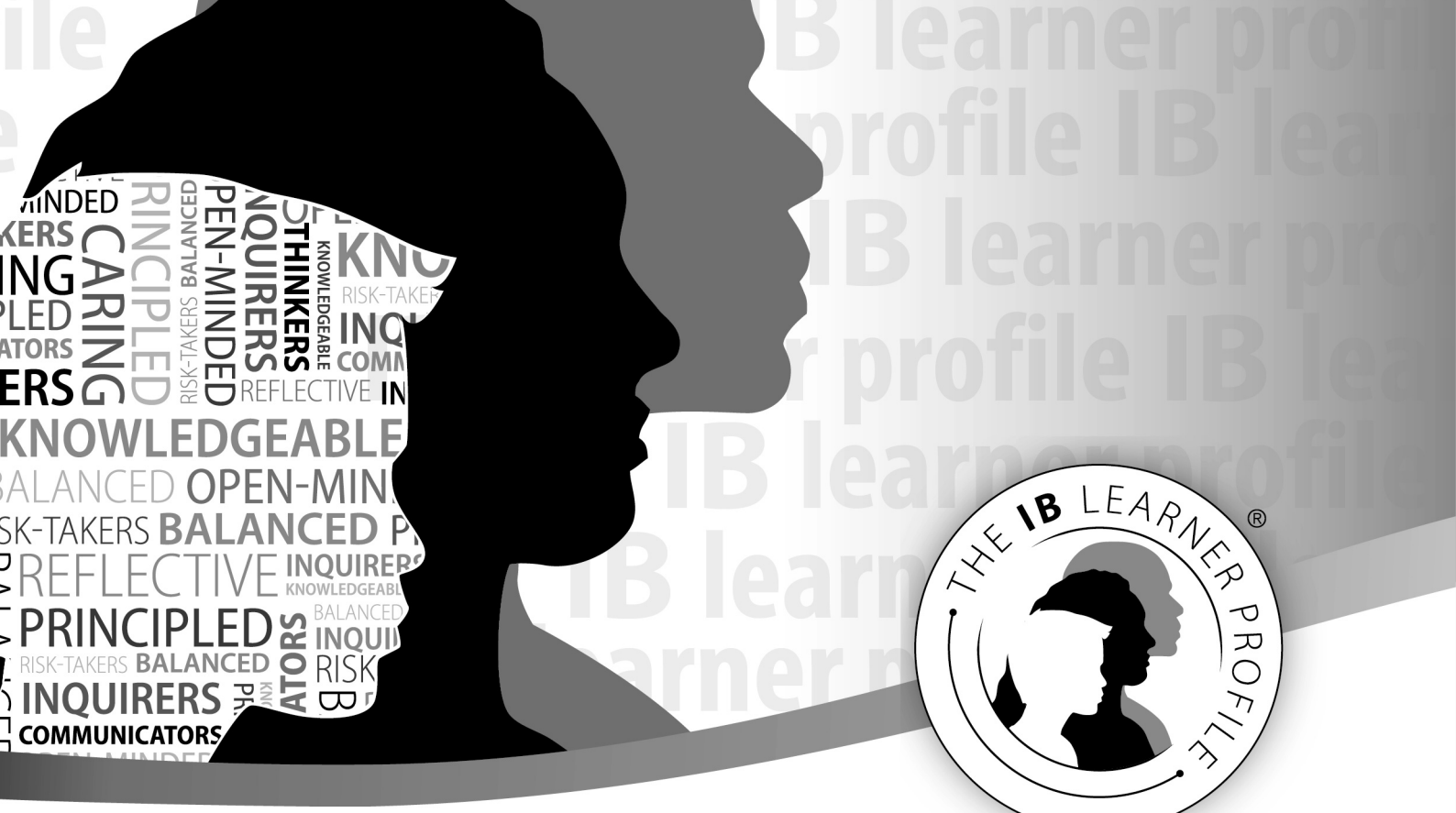
IB mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.



IB learner profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

As IB learners we strive to be:

INQUIRERS
We nurture our curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life.

KNOWLEDGEABLE
We develop and use conceptual understanding, exploring knowledge across a range of disciplines. We engage with issues and ideas that have local and global significance.

THINKERS
We use critical and creative thinking skills to analyse and take responsible action on complex problems. We exercise initiative in making reasoned, ethical decisions.

COMMUNICATORS
We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups.

PRINCIPLED
We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.

OPEN-MINDED
We critically appreciate our own cultures and personal histories, as well as the values and traditions of others. We seek and evaluate a range of points of view, and we are willing to grow from the experience.

CARING
We show empathy, compassion and respect. We have a commitment to service, and we act to make a positive difference in the lives of others and in the world around us.

RISK-TAKERS
We approach uncertainty with forethought and determination; we work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

BALANCED
We understand the importance of balancing different aspects of our lives—intellectual, physical, and emotional—to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live.

REFLECTIVE
We thoughtfully consider the world and our own ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development.

The IB learner profile represents 10 attributes valued by IB World Schools. We believe these attributes, and others like them, can help individuals and groups become responsible members of local, national and global communities.

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Contents

Purpose of this guide

This guide is for use from September 2014 or January 2015, depending on the start of the school year.

This document provides the framework for teaching and learning in mathematics in the Middle Years Programme (MYP) and must be read and used in conjunction with the document *MYP: From principles into practice* (May 2014), which includes:

- general information about the programme
- the MYP unit planner, with guidance for developing the curriculum that is relevant for all subject groups
- detailed information about approaches to learning
- advice that supports access and inclusion (including accommodations for students with learning support requirements
- a statement on academic honesty.

In MYP subject guides, requirements appear in a text box like this one.

Additional resources

Teacher support materials (TSMs) are available in the online curriculum centre (http://occ.ibo.org). The TSM for mathematics contains support for developing the written, taught and assessed curriculum. It provides examples of good practice including course overviews, assessment tasks and markschemes, as well as student work with teacher comments.

An optional process of externally-moderated assessment can lead to **IB MYP results** for mathematics courses, and these results can contribute to the awarding of an **IB MYP certificate**. More information is available in the annual publication *Handbook of procedures for the Middle Years Programme*.

A range of publications that support the MYP are available at the IB store (http://store.ibo.org).

Acknowledgments

The IB gratefully acknowledges the generous contributions of IB World Schools and a global community of educators who collaborate in the development of the Middle Years Programme.

Programme model

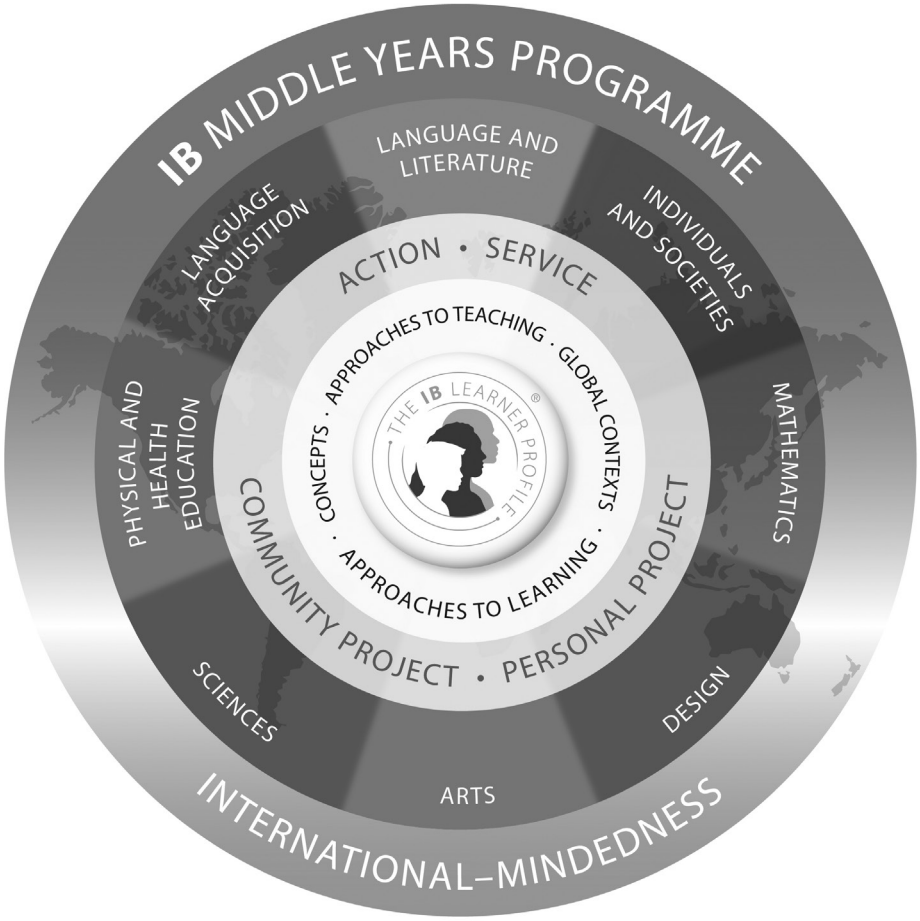

Figure 1
Middle Years Programme Model

The MYP is designed for students aged 11 to 16. It provides a framework of learning that encourages students to become creative, critical and reflective thinkers. The MYP emphasizes intellectual challenge, encouraging students to make connections between their studies in traditional subjects and the real world. It fosters the development of skills for communication, intercultural understanding and global engagement—essential qualities for young people who are becoming global leaders.

The MYP is flexible enough to accommodate the demands of most national or local curriculums. It builds upon the knowledge, skills and attitudes developed in the IB Primary Years Programme (PYP) and prepares students to meet the academic challenges of the IB Diploma Programme (DP) and the IB Career-related Certificate (IBCC).

The MYP

- addresses holistically students' intellectual, social, emotional and physical **well-being**
- provides students opportunities to develop the **knowledge**, **attitudes** and **skills** they need in order to manage complexity and take responsible action for the future
- ensures breadth and depth of understanding through study in **eight subject groups**
- requires the study of at least **two languages** to support students in understanding their own cultures and those of others
- empowers students to participate in **service with the community**
- helps to prepare students for **further education**, the **workplace** and a **lifetime of learning**.

Nature of mathematics

> Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of the world.
>
> Roger Bacon (1214–1294)

The study of mathematics is a fundamental part of a balanced education. It promotes a powerful universal language, analytical reasoning and problem-solving skills that contribute to the development of logical, abstract and critical thinking. Mathematics can help make sense of the world and allows phenomena to be described in precise terms. It also promotes careful analysis and the search for patterns and relationships, skills necessary for success both inside and outside the classroom. Mathematics, then, should be accessible to and studied by all students.

Studying mathematics, however, should be more than simply learning formulae or rules. Students should not have the impression that all of the answers to mathematics can be found in a book but, rather, that they can be active participants in the search for concepts and relationships. In that light, mathematics becomes a subject that is alive with the thrill of exploration and the rewards of discovery. At the same time, that new knowledge may then be applied to other situations, opening up even more doors for students. MYP mathematics promotes both inquiry and application, helping students to develop problem-solving techniques that transcend the discipline and that are useful in the world outside school.

An MYP mathematics programme should be tailored to the needs of students, seeking to intrigue and motivate them to want to learn its principles. Students should see authentic examples of how mathematics is useful and relevant to their lives and be encouraged to apply it to new situations. Mathematics provides the foundation for the study of sciences, engineering and technology. However, it is also evident in the arts and is increasingly important in economics, the social sciences and the structure of language. Students in the MYP are encouraged to use ICT tools to represent information, to explore and model situations, and to find solutions to various problems. These are skills that are useful in a wide range of arenas. MYP mathematics aims to equip all students with the knowledge, understanding and intellectual capabilities to address further courses in mathematics, as well as to prepare those students who will use mathematics in their studies, workplaces and lives in general.

Mathematics across the IB continuum

The IB continuum of international education provides a progression of learning for students aged 3–19. In the IB Primary Years Programme (PYP), mathematics is viewed primarily as a vehicle to support inquiry, providing a universal language through which we make sense of the world around us. It is intended that students become competent users of the language of mathematics and begin to use it as a way of thinking, as opposed to seeing mathematics as a series of facts and equations to be memorized. It is also recognized that students can appreciate the intrinsic fascination of mathematics and explore the world through its unique perceptions. In the same way that students describe themselves as "authors" or "artists", a school's mathematics programme should also provide students with the opportunity to see themselves as "mathematicians", who enjoy and are enthusiastic about exploring and learning about the subject. MYP mathematics aims to build on what students learn and do in the PYP and other student-centred programmes of primary education. There are no prior formal learning requirements.

In both the PYP and the MYP, it is important that learners acquire mathematical understanding by constructing their own meaning through increasing levels of abstraction, starting with an exploration of their own personal experiences, understandings and knowledge. Additionally, it is fundamental to the philosophy of both programmes that, since it is to be used in real-life situations, mathematics needs to be taught in relevant, realistic contexts, rather than by attempting to impart a fixed body of knowledge. In both programmes, mathematics is valued not only for its beauty but also for its usefulness in helping us to understand how the world works and for providing us with a unique way to communicate. Mathematics is an essential tool for transdisciplinary and interdisciplinary inquiry. Teaching and learning experiences in both the PYP and MYP challenge students to be curious, ask questions and explore and interact with the environment physically, socially and intellectually. Through engaging in this process, students are able to construct meaning about mathematics concepts, transfer this meaning to symbols and apply mathematical understanding in familiar and unfamiliar situations.

MYP mathematics courses help specifically to prepare students for the study of group 5 courses in the IB Diploma Programme (DP). As students progress from the MYP to the DP or IBCC, the emphasis on understanding increases as students work towards developing a strong mathematical knowledge base that will allow them to study a wide range of topics. Through this process they also work on communicating their ideas in ways that allow others to understand their thinking. The MYP mathematics objectives and criteria have been developed with both the internal and external assessment requirements of the DP in mind. The use of technology, which is required in DP mathematics, is also emphasized in the MYP as a tool for learning, applying and communicating mathematics. Where students in the MYP may select either standard or extended mathematics, the diploma mathematics programme offers four courses: mathematical studies standard level (SL), mathematics SL, mathematics higher level (HL) and further mathematics HL. MYP students enrolled in extended mathematics generally elect to take one of the HL mathematics courses in the Diploma Programme. Students in MYP standard mathematics should seek the recommendation of their teacher when deciding which course to pursue in the DP.

Figure 2 shows the IB continuum pathways to DP group 5 courses.

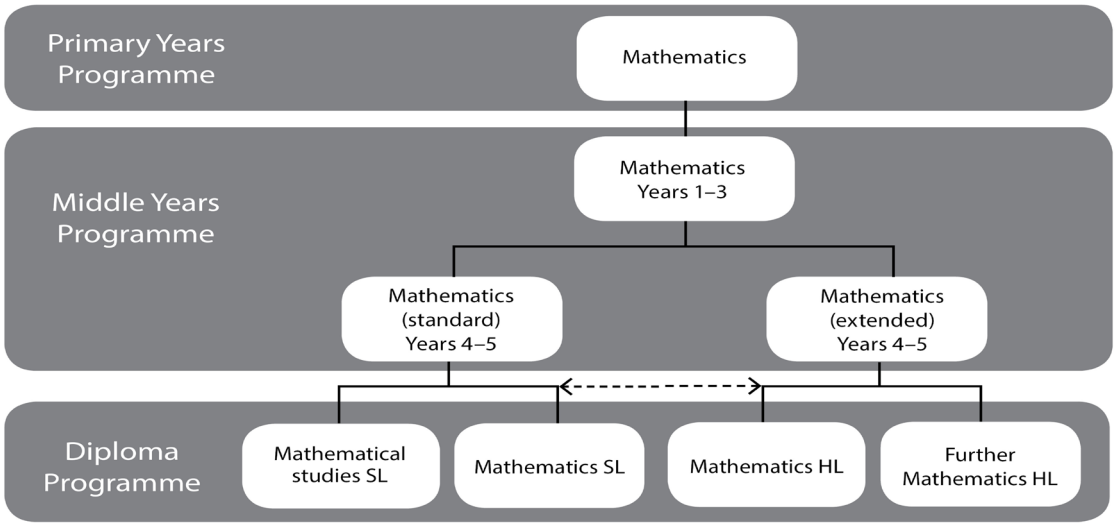

Figure 2

Pathway to Diploma Programme subjects—mathematical studies SL, mathematics SL, mathematics HL, and further mathematics HL

Regardless of the options available to students, possible general strategies teachers can use in developing a smooth transition between MYP and DP mathematics courses include:

- facilitating mathematics vertical planning sessions between the MYP and the DP
- developing an understanding and consistent use of a common set of key terms, notation and formulae that are applicable to all programmes
- preparing students to develop effective strategies for external examinations as well as inquiry-based learning across all mathematics courses
- providing students with the opportunity to explore problems that incorporate several areas of mathematics
- providing students with the opportunity to solve problems using mathematical concepts in unfamiliar situations.

The knowledge, skills and attitudes that students develop in mathematics courses provide a meaningful foundation for further study and help to prepare students for careers in, for example, climate research, actuary and insurance work, public-policy development, engineering, financial analysis and economic development, research and analysis, software development, biostatistics and epidemiology, law or medicine.

Aims

The aims of all MYP subjects state what a teacher may expect to teach and what a student may expect to experience and learn. These aims suggest how the student may be changed by the learning experience.

The aims of MYP mathematics are to encourage and enable students to:

- enjoy mathematics, develop curiosity and begin to appreciate its elegance and power
- develop an understanding of the principles and nature of mathematics
- communicate clearly and confidently in a variety of contexts
- develop logical, critical and creative thinking
- develop confidence, perseverance, and independence in mathematical thinking and problem-solving
- develop powers of generalization and abstraction
- apply and transfer skills to a wide range of real-life situations, other areas of knowledge and future developments
- appreciate how developments in technology and mathematics have influenced each other
- appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics
- appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives
- appreciate the contribution of mathematics to other areas of knowledge
- develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics
- develop the ability to reflect critically upon their own work and the work of others.

Objectives

The objectives of any MYP subject state the specific targets that are set for learning in the subject. They define what the student will be able to accomplish as a result of studying the subject.

The objectives of MYP mathematics encompass the factual, conceptual, procedural and metacognitive dimensions of knowledge.

Each objective is elaborated by a number of **strands**; a strand is an aspect or indicator of the learning expectation.

> Subject groups **must** address **all** strands of **all** four objectives **at least twice** in each year of the MYP.

The objectives for years 1, 3 and 5 of the programme are provided in the guide and their use is mandatory.

These objectives relate directly to the assessment criteria found in the "Assessed curriculum" section of this guide.

Together these objectives reflect the knowledge, skills and attitudes that students need in order to use mathematics in a variety of contexts (including real-life situations), perform investigations and communicate mathematics clearly.

A. Knowing and understanding

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop skills. This objective assesses the extent to which students can select and apply mathematics to solve problems in both familiar and unfamiliar situations in a variety of contexts.

> This objective requires students to demonstrate knowledge and understanding of the concepts and skills of the four branches in the prescribed framework (number, algebra, geometry and trigonometry, statistics and probability).

In order to reach the aims of mathematics, students should be able to:

i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations

ii. apply the selected mathematics successfully when solving problems

iii. solve problems correctly in a variety of contexts.

B. Investigating patterns

Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Working through investigations encourages students to become risk-takers, inquirers and critical thinkers. The ability to inquire is invaluable in the MYP and contributes to lifelong learning.

A task that does not allow students to select a problem-solving technique is too guided and should result in students earning a maximum achievement level of 6 (for years 1 and 2) and a maximum achievement level of 4 (for year 3 and up). However, teachers should give enough direction to ensure that all students can begin the investigation.

For year 3 and up, a student who describes a general rule consistent with incorrect findings will be able to achieve a maximum achievement level of 6, provided that the rule is of an equivalent level of complexity.

In order to reach the aims of mathematics, students should be able to:

i. select and apply mathematical problem-solving techniques to discover complex patterns

ii. describe patterns as general rules consistent with findings

iii. prove, or verify and justify, general rules.

C. Communicating

Mathematics provides a powerful and universal language. Students are expected to use appropriate mathematical language and different forms of representation when communicating mathematical ideas, reasoning and findings, both orally and in writing.

In order to reach the aims of mathematics, students should be able to:

i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations

ii. use appropriate forms of mathematical representation to present information

iii. move between different forms of mathematical representation

iv. communicate complete, coherent and concise mathematical lines of reasoning

v. organize information using a logical structure.

D. Applying mathematics in real-life contexts

MYP mathematics encourages students to see mathematics as a tool for solving problems in an authentic real-life context. Students are expected to transfer theoretical mathematical knowledge into real-world situations and apply appropriate problem-solving strategies, draw valid conclusions and reflect upon their results.

In order to reach the aims of mathematics, students should be able to:

i. identify relevant elements of authentic real-life situations

ii. select appropriate mathematical strategies when solving authentic real-life situations

iii. apply the selected mathematical strategies successfully to reach a solution

iv. justify the degree of accuracy of a solution

v. justify whether a solution makes sense in the context of the authentic real-life situation.

Planning a progression of learning

MYP mathematics relies on a progression in the complexity of the level of mathematics throughout the programme. For this reason, the objectives listed below for years 1, 3 and 5 are quite similar; however, the complexity of the mathematics being assessed is increasing.

Year 1 In order to reach the aims of mathematics, students should be able to:	Year 3 In order to reach the aims of mathematics, students should be able to:	Year 5 In order to reach the aims of mathematics, students should be able to:
Objective A: Knowing and understanding		
i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations ii. apply the selected mathematics successfully when solving problems iii. solve problems correctly in a variety of contexts.	i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations ii. apply the selected mathematics successfully when solving problems iii. solve problems correctly in a variety of contexts.	i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations ii. apply the selected mathematics successfully when solving problems iii. solve problems correctly in a variety of contexts.
Objective B: Investigating patterns		
i. apply mathematical problem-solving techniques to recognize patterns ii. describe patterns as relationships or general rules consistent with correct findings iii. verify whether the pattern works for other examples.	i. select and apply mathematical problem-solving techniques to discover complex patterns ii. describe patterns as relationships and/or general rules consistent with findings iii. verify and justify relationships and/or general rules.	i. select and apply mathematical problem-solving techniques to discover complex patterns ii. describe patterns as general rules consistent with findings iii. prove, or verify and justify, general rules.

Year 1 In order to reach the aims of mathematics, students should be able to:	Year 3 In order to reach the aims of mathematics, students should be able to:	Year 5 In order to reach the aims of mathematics, students should be able to:
Objective C: Communicating		
i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written statements ii. use different forms of mathematical representation to present information iii. communicate coherent mathematical lines of reasoning iv. organize information using a logical structure.	i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations ii. use appropriate forms of mathematical representation to present information iii. move between different forms of mathematical representation iv. communicate complete and coherent mathematical lines of reasoning v. organize information using a logical structure.	i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations ii. use appropriate forms of mathematical representation to present information iii. move between different forms of mathematical representation iv. communicate complete, coherent and concise mathematical lines of reasoning v. organize information using a logical structure.
Objective D: Applying mathematics in real-life contexts		
i. identify relevant elements of authentic real-life situations ii. select appropriate mathematical strategies when solving authentic real-life situations iii. apply the selected mathematical strategies successfully to reach a solution iv. explain the degree of accuracy of a solution v. describe whether a solution makes sense in the context of the authentic real-life situation.	i. identify relevant elements of authentic real-life situations ii. select appropriate mathematical strategies when solving authentic real-life situations iii. apply the selected mathematical strategies successfully to reach a solution iv. explain the degree of accuracy of a solution v. explain whether a solution makes sense in the context of the authentic real-life situation.	i. identify relevant elements of authentic real-life situations ii. select appropriate mathematical strategies when solving authentic real-life situations iii. apply the selected mathematical strategies successfully to reach a solution iv. justify the degree of accuracy of a solution v. justify whether a solution makes sense in the context of the authentic real-life situation.

Throughout the programme, students should be expected to demonstrate their understanding at increasing levels of sophistication.

The range of assessed skills, techniques and strategies as well as the complexity of their application, must increase as students progress through the programme.

Interdisciplinary learning

Interdisciplinary teaching and learning is grounded in individual subject groups and disciplines, but extends disciplinary understanding in ways that are:

- **integrative**—bringing together concepts, methods, or modes of communication from two or more subject groups, disciplines, or established areas of expertise to develop new perspectives
- **purposeful**—connecting disciplines to solve real-world problems, create products or address complex issues in ways that would have been unlikely through a single approach.

Interdisciplinary teaching and learning builds a connected curriculum that addresses the developmental needs of students in the MYP. It prepares students for further academic (inter)disciplinary study and for life in an increasingly interconnected world.

The MYP uses concepts and contexts as starting points for meaningful integration and transfer of knowledge across subject groups and disciplines. *Fostering interdisciplinary teaching and learning in MYP schools* (July 2014) contains more information, including a detailed process for planning and recording interdisciplinary units.

> MYP schools are responsible for engaging students in at least one collaboratively planned interdisciplinary unit for each year of the programme.

MYP mathematics offers many opportunities for interdisciplinary teaching and learning. Possible interdisciplinary units in this subject group could include inquiries into:

- collecting and analysing statistical data in physical and health education classes
- applying geometry knowledge in design projects
- investigating the links between musical theory and mathematical sequences.

Interdisciplinary learning can take place through both large and small-scale learning engagements. Authentic interdisciplinary learning often requires critical reflection and detailed collaborative planning. However, teachers and students can also make interdisciplinary connections through spontaneous learning experiences and conversations.

> All MYP subject group teachers are responsible for developing meaningful ongoing opportunities for interdisciplinary teaching and learning.

MYP projects

The MYP community project (for students in years 3 or 4) and MYP personal project (for students in year 5) aim to encourage and enable sustained inquiry within a global context that generates new insights and deeper understanding. In these culminating experiences, students develop confidence as principled, lifelong learners. They grow in their ability to consider their own learning, communicate effectively and take pride in their accomplishments.

Courses in mathematics help students to develop key approaches to learning (ATL) that lead to success and enjoyment in the MYP projects. In this subject group, students have important opportunities to practise ATL skills, especially cognitive skills. Organizing and transforming information are essential aspects of mathematics.

From their learning experiences in this subject group, students can find inspiration for their projects. Through the application of mathematics in real-life situations, students will be able to see a multitude of opportunities to incorporate their mathematical skills into the projects.

Mathematics offers many opportunities for learning through action. Inspiration from mathematics for community projects and personal projects might include inquiries into:

- the statistical analysis of a local or global sustainability issue
- the mathematical analysis of athletic performance by a team or individual
- developing networking solutions for transport routes to and from school.

Requirements

Teaching hours

Schools must allocate the teaching hours necessary to meet the requirements of MYP mathematics.

> The MYP requires at least 50 hours of teaching time for each subject area in each year of the programme.

In practice, more time is often necessary to meet subject area aims and objectives and to provide for the sustained, concurrent teaching that enables interdisciplinary study.

For students pursuing IB MYP results that can contribute to the awarding of the IB MYP certificate, mathematics courses should include at least 70 teaching hours in each of the final two years of the programme (MYP year 4 and MYP year 5).

Organizing mathematics in the school

All MYP subjects, including mathematics, provide a curricular framework with set final aims and objectives. MYP mathematics also provides a framework of content and skills organized into four branches, as seen below.

Levels of mathematics

MYP mathematics should be accessible to, and studied by, all students. Schools must ensure that the mathematics curriculum allows all students the opportunity to reach their full potential and achieve the final aims and objectives of MYP mathematics. Topics and skills of the framework for mathematics are organized so that students can work at two levels of challenge: **standard mathematics** and **extended mathematics**.

Standard mathematics aims to give all students a sound knowledge of basic mathematical principles while allowing them to develop the skills needed to meet the objectives of MYP mathematics.

Extended mathematics consists of the standard mathematics framework supplemented by additional topics and skills. This level provides the foundation for students who wish to pursue further studies in mathematics: for example, mathematics higher level (HL) as part of the IB Diploma Programme. Extended mathematics provides greater breadth and depth to the standard mathematics framework.

A common approach to implementing both levels is to have separate classes for standard mathematics and extended mathematics during the final two years of the MYP. In MYP years 1 to 3, students often take a common differentiated mathematics course or pursue an accelerated course sequence.

Within the prescribed framework, all MYP mathematics courses should ensure that students:

- apply mathematics to authentic real-life situations
- perform investigations to discover patterns.

Planning the mathematics curriculum

IB World Schools are responsible for developing and structuring MYP mathematics courses that provide opportunities for students to meet the aims and objectives of the programme. Each school's circumstances, including local and national curriculum requirements, determine the organization of mathematics within the school.

The MYP requires schools to facilitate and promote collaborative planning for the purpose of curriculum development and review.

Mathematics objectives for years 1 to 5 of the curriculum provide continuity and outline a progression of learning. These objectives guide teachers in making decisions about developmentally-appropriate learning experiences, including formative and summative assessments.

As they develop the vertical articulation of mathematics over the years of the programme, teachers should plan increasingly complex units of work that encompass multiple objectives. However, within these units, discrete tasks or smaller units of work might concentrate on specific objectives or individual strands.

Mathematics courses offer many opportunities to build interdisciplinary connections across the curriculum. Horizontal articulation for each year of the programme should coordinate teaching and learning across courses in mathematics, as well as identify shared conceptual understandings and approaches to learning that span multiple subject groups and help to create a coherent learning experience for students throughout the year.

In mathematics, learning is generally sequential. Success in later mathematics courses relies on building fundamentals in earlier ones. Not only content but also teaching methods, assessments and problem-solving strategies need to be sequenced appropriately. Teachers are encouraged to articulate the mathematics continuum in their schools. This may be done in a number of ways; however, a planned approach is necessary. Consideration of concepts, skills and processes is required to align learning and assessment vertically. The following examples demonstrate possible ways that activities can be sequenced to prepare students for future DP mathematics content. These examples demonstrate different approaches teachers could take when planning a smooth transition from MYP year 1 through to mathematics in the DP.

Example 1: Modelling anthropometric (body-part) measurements

This example demonstrates a method that could be used to sequence skills required for an internal assessment. This example focuses on activities to model anthropometric measurements (measurement of the human individual) from MYP year 1 to the DP.

Year	Skills	Possible activities
MYP 1	• Cartesian plane • Plotting points • Relating variables • Data collection • Informal and formal description of observed trend	Simple models, such as height of person versus length of foot
MYP 3	• Trends and predictions • Modelling "by eye" • Independent versus dependent variables • Appropriate values for variables	Extension of modelling data: for example, male versus female
MYP 5	• Domain and range • Linear regression • Accuracy (correlation) • Prediction • Use of technology	Modelling-based extensions of the domain and range: for example, height (male or female) versus length of foot (male or female) Solving crimes: If you have the length of the foot, how tall is the person likely to be? Can giants exist?
DP 1–2	• Choosing own models • Comparing • Correlation coefficient • Use of more complex models • Students discuss limitations of their models and data	Modelling based on a variety of functions Application of paleo-anthropometry (relating sizes of bones to determine the height of a dinosaur/pre-human)

Example 2: Volume and area

This example demonstrates a method that could be used for sequencing a specific problem. This example focuses on activities to investigate volume and area from MYP year 1 to the DP.

Year	Possible activities
MYP 1	Students will investigate the volume of different cylindrical containers by taking appropriate measurements and comparing their estimated value to the calculated value. They will produce a report hypothesizing why there may be differences and why certain types of containers are used for certain products.
MYP 3	Students will investigate the effects of changing the dimensions (radius and height) of a cylindrical container and produce a report describing which dimension has the greatest effect on surface area and volume.
MYP 5	Students will determine the dimensions of a can of soft drink that will minimize its surface area given a fixed volume (330 ml) through the use of graphing (using a graphic display calculator (GDC)).
DP 1–2	Students will determine the dimensions of a can of soft drink that will minimize its surface area given a fixed volume (330 ml) through the use of calculus and compare it to the value produced using another method.

Work that can be done to ease the transition from MYP to DP in any of these areas will help students be more successful and the previous examples need not be construed as the only means of accomplishing this.

Teaching and learning through inquiry

Inquiry, in the broadest sense, is the process that people use to move to deeper levels of understanding. Inquiry involves speculating, exploring, questioning and connecting. In all IB programmes, inquiry develops curiosity and promotes critical and creative thinking.

The MYP structures sustained inquiry in mathematics by developing **conceptual understanding** in **global contexts**. Teachers and students develop a **statement of inquiry** and use **inquiry questions** to explore the subject. Through their inquiry, students develop specific interdisciplinary and disciplinary **approaches to learning**.

Conceptual understanding

A concept is a "big idea"—a principle or notion that is enduring, the significance of which goes beyond particular origins, subject matter or place in time. Concepts represent the vehicle for students' inquiry into the issues and ideas of personal, local and global significance, providing the means by which they can explore the essence of mathematics.

Concepts have an important place in the structure of knowledge that requires students and teachers to think with increasing complexity as they organize and relate facts and topics.

Concepts express understanding that students take with them into lifelong adventures of learning. They help students to develop principles, generalizations and theories. Students use conceptual understanding as they solve problems, analyse issues and evaluate decisions that can have an impact on themselves, their communities and the wider world.

> In the MYP, conceptual understanding is framed by prescribed key and related concepts. Teachers must use these concepts to develop the curriculum. Schools may identify and develop additional concepts to meet local circumstances and curriculum requirements.

Key concepts

Key concepts promote the development of a broad curriculum. They represent big ideas that are relevant both within and across disciplines and subjects. Inquiry into key concepts can facilitate connections between and among:

- courses within the mathematics subject group (intra-disciplinary learning)
- other subject groups (interdisciplinary learning).

Table 1 lists the key concepts to be explored across the MYP. The key concepts contributed by the study of mathematics are **form**, **logic** and **relationships**.

Aesthetics	Change	Communication	Communities
Connections	Creativity	Culture	Development
Form	Global interactions	Identity	**Logic**
Perspective	**Relationships**	Time, place and space	Systems

Table 1
MYP key concepts

These key concepts provide a framework for mathematics, informing units of work and helping to organize teaching and learning.

Concept is **form**

Form is the shape and underlying structure of an entity or piece of work, including its organization, essential nature and external appearance.

Form in MYP mathematics refers to the understanding that the underlying structure and shape of an entity is distinguished by its properties. Form provides opportunities for students to appreciate the aesthetic nature of the constructs used in a discipline.

Concept is **logic**

Logic is a method of reasoning and a system of principles used to build arguments and reach conclusions.

Logic in MYP mathematics is used as a process in making decisions about numbers, shapes, and variables. This system of reasoning provides students with a method for explaining the validity of their conclusions. Within the MYP, this should not be confused with the subfield of mathematics called "symbolic logic".

Concept is **relationships**

Relationships allow students to identify and understand connections and associations between properties, objects, people and ideas—including the human community's connections with the world in which we live. Any change in relationships brings consequences—some of which may occur on a small scale, while others may be far-reaching, affecting large systems like human societies and the planet as a whole.

Relationships in MYP mathematics refers to the connections between quantities, properties or concepts and these connections may be expressed as models, rules or statements. Relationships provide opportunities for students to explore patterns in the world around them. Connections between the student and mathematics in the real world are important in developing deeper understanding.

Other key concepts can also be important in mathematics and these are outlined in the **Mathematics skills framework** section of this guide.

Related concepts

Related concepts promote deep learning. They are grounded in specific disciplines and are useful for exploring key concepts in greater detail. Inquiry into related concepts helps students develop more complex and sophisticated conceptual understanding. Related concepts may arise from the subject matter of a unit or from the craft of a subject—that is, its features and processes.

Table 2 lists related concepts for the study of mathematics.

Related concepts in mathematics		
Change	Equivalence	Generalization
Justification	Measurement	Model
Pattern	Quantity	Representation
Simplification	Space	System

Table 2
Related concepts in mathematics

The appendices contain a glossary of these related concepts for mathematics.

Global contexts for teaching and learning

Global contexts direct learning towards independent and shared inquiry into our common humanity and shared guardianship of the planet. Using the world as the broadest context for learning, MYP mathematics can develop meaningful explorations of:

- identities and relationships
- orientation in space and time
- personal and cultural expression
- scientific and technical innovation
- globalization and sustainability
- fairness and development

Teachers must identify a global context for teaching and learning, or develop additional contexts that help students explore the relevance of their inquiry (why it matters).

Many inquiries into mathematics concepts naturally focus on scientific and technical innovation. However, courses in this subject group should, over time, offer students multiple opportunities to explore all MYP global contexts in relation to the aims and objectives of the subject group.

Statements of inquiry

Statements of inquiry set conceptual understanding in a global context in order to frame classroom inquiry and direct purposeful learning. Table 3 shows some possible statements of inquiry for MYP mathematics units.

Statement of inquiry	Key concept Related concepts Global context	Possible project/study
Architects and engineers must use finite resources responsibly when they design new structures.	• Form • Space • Quantity • Fairness and development	Geometry and trigonometry—volume
Logic is a powerful tool for justifying what we discover through measurement and observation.	• Logic • Measurement • Justification • Orientation in space and time	Geometry and trigonometry—parallel lines and transversals
Decision-making can be improved by using a model to represent relationships.	• Relationships • Model • Representation • Identities and relationships	Algebra—quadratic functions
Understanding form and shape enhances creativity.	• Form • Pattern • Space • Personal and cultural expression	Geometry and trigonometry—transformations
Modelling using a logical process helps us to understand the world.	• Logic • Pattern • Simplification • Model • Scientific and technical innovation	Algebra—projectile motion
Discovering mathematical relationships can lead to a better understanding of how environmental systems evolve.	• Relationships • System • Change • Globalization and sustainability	Number—exponentials and logarithms
Establishing patterns in the natural world can help in understanding relationships.	• Relationships • Pattern • Scientific and technical innovation	Statistics and probability—line of best fit

Table 3
Example statements of inquiry

Inquiry questions

Teachers and students use statements of inquiry to help them identify factual, conceptual and debatable inquiry questions. Inquiry questions give direction to teaching and learning, and they help to organize and sequence learning experiences.

Table 4 shows some possible inquiry questions for MYP mathematics units.

Factual questions: Remembering facts and topics	Conceptual questions: Analysing big ideas	Debatable questions: Evaluating perspectives and developing theories
• How do the gradients of perpendicular lines compare? • How does the volume of a quantity differ from its area? • What determines whether two events are independent?	• What does it mean to have a "solution" of a function? • Why can estimation be useful? • How could we map the neural network of a human brain?	• What is more natural: order or chaos? • Are all events in the universe determined by probability? • How big is infinity?

Table 4

Examples of factual, conceptual and debatable questions

Approaches to learning

All MYP units of work offer opportunities for students to develop and practise **approaches to learning** (ATL) skills. ATL skills provide valuable support for students working to meet the subject group's aims and objectives.

ATL skills are grouped into five categories that span the IB continuum of international education. IB programmes identify discrete skills in each category that can be introduced, practised and consolidated in the classroom and beyond.

While ATL skills are relevant across all MYP subject groups, teachers may also identify ATL skill indicators especially relevant for, or unique to, a particular subject group or course.

Table 5 suggests some of the indicators that can be important in mathematics.

Category	Skill indicator
Thinking skills	Use prioritization and order of precedence in problem-solving
Social skills	Help others to create success for themselves during group work
Communication skills	Organize and interpret data using both analogue and digital tools
Self-management skills	Practise focus and concentration while solving multiple problems
Research skills	Use a variety of technologies and media platforms, including social media and online networks, to source information

Table 5

Examples of mathematics-specific skill indicators

Well-designed learning engagements and assessments provide rich opportunities for students to practise and demonstrate ATL skills. Each MYP unit explicitly identifies ATL skills around which teaching and learning can focus and through which students can authentically demonstrate what they are able to do. Formative assessments provide important feedback for developing discrete skills, and many ATL skills support students as they demonstrate their achievements in summative assessments of subject group objectives.

Table 6 lists some specific ATL skills that students can demonstrate through performances of understanding in mathematics.

Approaches to learning
Thinking (critical thinking): draw justifiable conclusions and generalizations from investigating patterns **Communication (reflection):** keep a regular journal during the investigation to maintain a record of reflections

Table 6

Examples of demonstrations of ATL skills in mathematics

Mathematics skills framework

The framework for MYP mathematics outlines four branches of mathematical study.

- Number
- Algebra
- Geometry and trigonometry
- Statistics and probability

Schools can use the framework for mathematics as a tool for curriculum mapping when designing and planning their mathematics courses. **Schools are not expected to address all the branches of the framework in each year of the programme, nor are they required to teach every topic or skill suggested in the framework.** However, over the five years (or complete duration) of the programme, students should experience learning in all four branches of the framework for mathematics.

The topics and skills are **examples** of what students may expect to study at the two levels—standard mathematics and extended mathematics. A topic listed as "extended" in the framework could be a topic in a standard mathematics class in some cases. Schools are responsible for defining the distinction between standard and extended mathematics courses.

Extended mathematics courses effectively prepare students for advanced study when they feature depth and complexity of key topics, develop independent mathematical problem-solving, and extend students' mathematical knowledge and skills to other applications.

Number

The ability to work with numbers is an essential skill in mathematics. Students are expected to have an understanding of number concepts and to develop the skills of calculation and estimation. Students should understand that the use of numbers to express patterns and to describe real-life situations goes back to humankind's earliest beginnings, and that mathematics has multicultural roots.

Links to MYP concepts

Key concepts from other MYP subjects that could be used within the number branch include **change** (ratios, number bases), **communication** (number lines, units of measurement), **connections** (number bases, number sequences, Venn diagrams), **development** (number sequences, prime numbers), **identity** (sets, factors) and **systems** (sets, number systems). Related concepts from MYP mathematics that could be used within the **number** branch include **equivalence**, **generalization**, **justification**, **measurement**, **quantity**, **simplification** and **system**.

Topic	Skills
Standard and extended mathematics	
Forms of numbers: integers, fractions, decimals, exponents, absolute value, standard form (scientific notation), recurring decimals and surds/ radicals Number systems: set of positive integers and zero (N), integers (Z), rational numbers (Q), irrational numbers (Q'), and real numbers (R)	Ordering numbers Absolute value of a number Transformation between different forms of numbers Simplification of numerical expressions in the number systems and forms of number Recognizing and classifying numbers in different number systems, including recurring decimals
Sets Venn diagrams	Basic vocabulary (element, subset, null set, and so on) Performing operations Properties of sets (commutative, associative, distributive) Drawing and interpreting Venn diagrams Using Venn diagrams to solve problems in real-life contexts
The four number operations	Using the four number operations (addition, subtraction, multiplication and division) with integers, decimals and fractions
Prime numbers and factors, including greatest common divisor and least common multiple	Representing a number as the product of its prime factors and using this representation to find the greatest common divisor and least common multiple
Number lines	Expressing the solution set of a linear inequality on the number line (as well as set notation)
Estimation	Using different forms of rounding; decimal approximation and significant figures Using appropriate forms of rounding to estimate results
Units of measurement	Converting between different units of measurement and between different currencies
Ratio, percentage; direct and inverse proportion	Dividing a quantity in a given ratio Finding a constant of proportionality, setting up equations and graphing direct and inverse relationships
Number sequences	Predicting the next term in a number sequence (linear, quadratic, triangular, Fibonacci)
Integer exponents	Evaluating numbers with integer exponents

Topic	Skills
Extended mathematics	
Fractional exponents	Using the rules of indices to simplify numerical expressions involving radicals and exponents
Logarithms	Evaluating the logarithm of a number and simplifying numerical expressions
Number bases	Performing operations with numbers in different bases

Algebra

Algebra is an abstraction of the concepts first used when dealing with number and is essential for further learning in mathematics. Algebra uses letters and symbols to represent numbers, quantities and operations, and employs variables to solve mathematical problems.

Students who wish to continue studying mathematics beyond the MYP will require knowledge of concepts and skills in algebra. Teachers should assist students' understanding of algebra by using real-life contexts for the application of algebraic knowledge and skills in problem-solving situations. To develop deeper problem-solving understanding, algebra topics can be linked to modelling, representations and connections.

Links to MYP concepts

Key concepts from other MYP subjects that could be used within the **algebra** branch include **aesthetics** (patterns and sequences, graphs), **change** (algebraic expressions, transformations), **connections** (patterns and sequences, functions and graphs), **systems** (functions, series), and **time, place, and space** (functions, equations). Related concepts from MYP mathematics that could be used within the **algebra** branch include **change**, **equivalence**, **pattern**, **quantity**, **representation**, **simplification**, and **system**.

Topic	Skills
Standard and extended mathematics	
Addition, subtraction, multiplication and division of algebraic terms	Expanding and simplifying algebraic expressions
Factorization of algebraic expressions	Factorizing linear and quadratic expressions
Substitution	Using substitution to evaluate expressions
Rearranging algebraic expressions	Changing the subject of the formula
Algebraic fractions	Solving equations involving algebraic fractions
Integer and fractional exponents (including negative number exponents)	Using the laws of exponents
Patterns and sequences	Finding and justifying or proving general rules/ formulae for sequences
Algorithms	Analysing and using well-defined procedures for solving complex problems

Topic	Skills
Functions • Types of functions: linear, quadratic, exponential, sine and cosine • Domain and range • Transformations	The linear function, $f(x) = mx + c$, its graph, gradient and y-intercept Parallel and perpendicular lines and the relationships between their gradients Describing transformed linear, quadratic, exponential, and sine and cosine functions Example: $f(x) = a(x - h)^2 + k$ Note: Sine and cosine functions are limited to the form $f(x) = a\sin(bx) + c$ Graphing different types of functions and understanding their characteristics Determining the range, given the domain Translating, reflecting and dilating functions
Equations: • Linear • Quadratic • Simultaneous	Solving equations algebraically and graphically
Inequalities	Solving and graphing linear inequalities Linear programming
Extended mathematics	
Logarithms with different base number (including natural logarithms)	Using the laws of logarithms
Functions and graphs • Sine and cosine, logarithmic and rational (of the form $f(x) = 1/x$) functions • Inverse and composite functions	Graphing different types of functions and understanding their characteristics Addition and subtraction of functions Determining inverse and composite functions and their graphs Solving equations algebraically and graphically
Inequalities	Solving non-linear inequalities
Transformations of functions	Describing and analysing transformed logarithmic, rational (of the form $f(x) = 1/x$), and sine and cosine functions Example: $f(x) = a\sin(bx - c) + d$
Arithmetic and geometric series	Developing, and justifying or proving, general rules/formulae for sequences Finding the sum of the series, including infinite series

Geometry and trigonometry

The study of geometry and trigonometry enhances students' spatial awareness and provides them with the tools for analysing, measuring and transforming geometric quantities in two and three dimensions.

Links to MYP concepts

Key concepts from other MYP subjects that could be used within the **geometry and trigonometry** branch include **aesthetics** (geometric shapes, transformations), **change** (identities, transformations), **communities** (angle properties, triangle properties), **creativity** (transformations, similarity and congruency), **identity** (unit circle, identities), **perspective** (coordinate geometry, similarity and congruency), and **time, place and space** (three-dimensional coordinate geometry, transformations). Related concepts from MYP mathematics that could be used within the **geometry and trigonometry** branch include **change**, **equivalence**, **model**, **pattern**, **quantity**, **representation**, **space** and **system**.

Topic	Skills
Standard and extended mathematics	
Geometrical elements and their classification	Naming and classifying different geometrical elements (point, line, plane, angle, regular and irregular planar figures, solids)
Distance	Measuring the distance between two points, and between a line and a point
Angle properties	Solving problems using the properties of: • angles in different figures or positions • acute, right and obtuse angles in triangles • angles in intersecting and parallel lines • angles in regular and irregular polygons • angles in circles
Triangle properties	Solving problems involving triangles by using: • Pythagoras' theorem and its converse • properties of similar triangles • properties of congruent triangles
Perimeter/area/volume	Finding the perimeter (circumference), area and volume of regular and irregular two-dimensional (2D) and three-dimensional (3D) shapes Compound shapes
The Cartesian plane	Identifying the different components of the Cartesian plane: axes, origin, coordinates *(x, y)* and points Understanding and using the Cartesian plane, plotting graphs and finding distances between points, finding the midpoint

Topic	Skills
Trigonometric ratios in right-angled triangles	Relating angles and sides of right-angled triangles using sine, cosine and tangent Solving problems in right-angled triangles using trigonometric ratios
Simple isometric transformations	Transforming a figure by rotation, reflection, translation and enlarging
Circle geometry	Using circle theorems to find: • lengths of chords • measures of angles and arcs • perimeter and area of sectors
Extended mathematics	
Three-dimensional coordinate geometry	Distance, section and midpoint formulae
Similarity and congruency	Justifying and proving using theorems of similarity and congruency
Vectors and vector spaces	Addition, subtraction and scalar multiplication of vectors, both algebraically and graphically Dot product
Sine and cosine rules	Using the sine and cosine rules to solve problems
Trigonometric identities	Using simple trigonometric identities to simplify expressions and solve equations where $0° \leq \theta \leq 360°$. Note: Simple trigonometric identities expected are: $\sin^2(x) + \cos^2(x) = 1$ $\tan(x) = \sin(x)/\cos(x)$
Angle measures	Converting angles between degrees and radians Using radians to solve problems, where appropriate
The unit circle	Finding the exact value of trigonometric functions of special angles

Note: Radians are not required in standard mathematics.

Statistics and probability

This branch of mathematics is concerned with the collection, analysis and interpretation of quantitative data and uses the theory of probability to estimate parameters, discover empirical laws, test hypotheses and predict the occurrence of events.

Through the study of statistics, students should develop skills associated with the collection, organization and analysis of data, enabling them to present information clearly and to discover patterns. Students will also develop critical-thinking skills, enabling them to differentiate between what happens in theory (probability) and what is observed (statistics).

Students should understand both the power and limitations of statistics, becoming aware of their legitimate use in supporting and questioning hypotheses, but also recognizing how statistics can be used to mislead as well as to counter opinions and propaganda.

Students should use these skills in their investigations and are encouraged to use information and communication technology (ICT) whenever appropriate.

Links to MYP concepts

Key concepts from other MYP subjects that could be used within the **statistics and probability** branch include **communication** (representation, probability of events), **communities** (samples, populations), **connections** (probability of successive trials, measures of central tendency), **development** (probability of successive trials, population sampling), **global interaction** (population sampling, representations) and **systems** (probability of events, conditional probability). Related concepts from MYP mathematics that could be used within the **statistics and probability** branch include **change**, **equivalence**, **generalization**, **justification**, **measurement**, **model**, **pattern**, **quantity**, **representation**, **simplification** and **system**.

Topic	Skills
Standard and extended mathematics	
Graphical analysis and representation (pie charts, histograms, line graphs, scatter plots, box-and-whisker plots)	Data collection Constructing and interpreting graphs Drawing the line of best fit
Population sampling	Selecting samples and making inferences about populations
Measures of central tendency/location (mean, mode, median, quartile, percentile) for discrete and continuous data	Calculating the mean, median and mode, and choosing the best measure of central tendency
Measures of dispersion (range, interquartile range) for discrete and continuous data	Calculating the interquartile range

Topic	Skills
Probability of an event Probability of independent, mutually exclusive and combined events Probability of successive trials	Calculating probabilities of simple events, with and without replacement Calculating probabilities of independent events, mutually exclusive events and combined events Solving problems using tree diagrams and Venn diagrams
Extended mathematics	
Standard deviation	Making inferences about data given the mean and standard deviation
Conditional probability	Calculating conditional probability

Subject-specific guidance

Effective use of information and communication technology (ICT) in mathematics

The appropriate use of computers, computer applications and calculators can improve the understanding of all students. Depending upon the school resources, ICT should be used whenever appropriate:

- as a means of expanding students' knowledge of the world in which they live
- as a channel for developing concepts and skills
- as a powerful communication tool.

ICT provides a wide range of resources and applications for teachers to explore in order to enhance teaching and learning. In mathematics, ICT can be used as a tool to perform complicated calculations, solve problems, draw graphs, and interpret and analyse data. ICT can also be helpful to

- investigate data and mathematical concepts
- obtain rapid feedback when testing out solutions
- observe patterns and make generalizations
- move between analytical and graphical representations
- visualize geometrical transformations.

In addition, the appropriate use of ICT can enhance students' communication skills, assisting them in the collection, organization and analysis of information and in the presentation of their findings.

However, for ICT to be a useful tool for learning, students need to be familiar with the resources and applications, and know how and when to use them. Students should be able to decide when the use of ICT is appropriate and when alternative methods such as pencil and paper, mental calculation or diagrams should be used. Therefore, it is important that teachers show students how to use these resources effectively while supporting the development of their intellectual skills.

ICT can support students with special educational needs who have difficulties understanding a particular concept or who would benefit from further practice. It can also provide the extra challenge for gifted and talented students to explore further ideas and concepts. "Adaptive technologies" can enable students with severe learning disabilities to become active learners in the classroom alongside their peers. For more information about adaptive technologies and special educational needs (SEN), please refer to the SEN page on the online curriculum centre (OCC).

Depending on the school facilities and the availability of ICT resources, teachers are encouraged to use ICT whenever possible and appropriate as a means of enhancing learning.

Some of the possible ICT resources in mathematics might include:

- databases and spreadsheets
- graph-plotter software

- dynamic geometry software
- mathematics content-specific software
- graphic display calculators (GDCs)
- internet search engines
- CD-ROMs
- word processing or desktop publishing
- graphic organizers
- Computer Algebra System (CAS).

Assessment tasks in MYP mathematics

Generally, criteria A, B and D are assessed with different kinds of tasks. Criterion C is often used to assess constructed responses and reports in combination with criterion B or D.

Criterion	Typical assessment tasks	Notes
Criterion A (knowing and understanding)	Classroom tests Examinations Assignments that include both familiar and unfamiliar situations	Teachers who choose to use criterion A with criterion B should be able to clearly define which aspects of the task will be assessed with each criterion, ensuring that the task is rigorous enough to allow students to achieve the highest levels of both criteria.
Criterion B (investigating patterns)	Mathematical investigations of some complexity that allow students: • to choose their own mathematical techniques • to reason from the specific to the general	Assessment tasks could have a variety of solutions and should enhance independent mathematical thinking.

Criterion	Typical assessment tasks	Notes
Criterion C (communicating)	Investigations and real-life problems Reports that: • require logical structure • allow multiple forms of representation to present information	Tests are not appropriate to assess this criterion since there is no logical structure to the whole piece of work that is submitted by the student. Criterion C is used when students present a report, for example, that requires a logical structure in order to be followed and that would allow for several forms of representation to be used to present information.
Criterion D (applying mathematics in real-life contexts)	Opportunities to use mathematical concepts to solve real-life problems	For example: modelling or curve-fitting tasks based in authentic contexts. Mathematics can be used to model many situations (for example, painting a room, analysing mobile telephone tariff plans, triangulation, diet plans).

Alignment of objectives and criteria

In the MYP, assessment is closely aligned with the written and taught curriculum. Each strand from MYP mathematics has a corresponding strand in the assessment criteria for this subject group. Figure 3 illustrates this alignment and also the increasingly complex demands for student performance at higher levels of achievement.

A: Knowing and understanding

At the end of year 5, students should be able to:

i. **select** appropriate mathematics when solving problems in both familiar and unfamiliar situations

ii. **apply** the selected mathematics successfully when solving problems

iii. **solve** problems correctly in a variety of contexts.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1-2	The student is able to: i. **select** appropriate mathematics when solving simple problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
3-4	The student is able to: i. **select** appropriate mathematics when solving more complex problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
5-6	The student is able to: i. **select** appropriate mathematics when solving challenging problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
7-8	The student is able to: i. **select** appropriate mathematics when solving challenging problems in both familiar and unfamiliar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.

Figure 3
Mathematics objectives and criteria alignment

Assessment criteria overview

Assessment for mathematics courses in all years programme is criterion-related, based on four equally weighted assessment criteria:

Criterion A	Knowing and understanding	**Maximum 8**
Criterion B	Investigating patterns	**Maximum 8**
Criterion C	Communicating	**Maximum 8**
Criterion D	Applying mathematics in real-life contexts	**Maximum 8**

Subject groups **must** address **all** strands of **all** four assessment criteria **at least twice** in each year of the MYP.

In the MYP, subject group objectives correspond to assessment criteria. Each criterion has nine possible levels of achievement (0–8), divided into four bands that generally represent limited (1–2); adequate (3–4); substantial (5–6); and excellent (7–8) performance. Each band has its own unique descriptor that teachers use to make "best-fit" judgments about students' progress and achievement.

This guide provides the **required assessment criteria** for years 1, 3 and 5 of MYP mathematics. In response to national or local requirements, schools may add criteria and use additional models of assessment. Schools must use the appropriate assessment criteria, as published in this guide, to report students' final achievement in the programme.

Teachers clarify the expectations for each summative assessment task with direct reference to these assessment criteria. Task-specific clarifications should clearly explain what students are expected to know and do. They might be in the form of:

- a task-specific version of the required assessment criteria
- a face-to-face or virtual classroom discussion
- a detailed task sheet or assignment.

Mathematics assessment criteria: Year 1

Criterion A: Knowing and understanding

Maximum: 8

At the end of year 1, students should be able to:

i. **select** appropriate mathematics when solving problems in both familiar and unfamiliar situations

ii. **apply** the selected mathematics successfully when solving problems

iii. **solve** problems correctly in a variety of contexts.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **select** appropriate mathematics when solving simple problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
3–4	The student is able to: i. **select** appropriate mathematics when solving more complex problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
5–6	The student is able to: i. **select** appropriate mathematics when solving challenging problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
7–8	The student is able to: i. **select** appropriate mathematics when solving challenging problems in both familiar and unfamiliar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.

Criterion B: Investigating patterns

Maximum: 8

At the end of year 1, students should be able to:

i. **apply** mathematical problem-solving techniques to recognize patterns

ii. **describe** patterns as relationships or general rules consistent with correct findings

iii. **verify** whether the pattern works for other examples.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **apply**, with teacher support, mathematical problem-solving techniques to recognize simple patterns ii. **state** predictions consistent with simple patterns.
3–4	The student is able to: i. **apply** mathematical problem-solving techniques to recognize patterns ii. **suggest** how these patterns work.
5–6	The student is able to: i. **apply** mathematical problem-solving techniques to recognize patterns ii. **suggest** relationships or general rules consistent with findings iii. **verify** whether patterns work for another example.
7–8	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to recognize correct patterns ii. **describe** patterns as relationships or general rules consistent with correct findings iii. **verify** whether patterns work for other examples.

Note: A task that does not allow students to select a problem-solving technique is too guided and should result in students earning a maximum achievement level of 6 (for years 1 and 2).

Criterion C: Communicating

Maximum: 8

At the end of year 1, students should be able to:

i. **use** appropriate mathematical language (notation, symbols and terminology) in both oral and written statements

ii. **use** different forms of mathematical representation to present information

iii. **communicate** coherent mathematical lines of reasoning

iv. **organize** information using a logical structure.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **use** limited mathematical language ii. **use** limited forms of mathematical representation to present information iii. **communicate** through lines of reasoning that are difficult to understand.
3–4	The student is able to: i. **use** some appropriate mathematical language ii. **use** different forms of mathematical representation to present information adequately iii. **communicate** through lines of reasoning that are able to be understood, although these are not always coherent iv. adequately **organize** information using a logical structure.
5–6	The student is able to: i. usually **use** appropriate mathematical language ii. usually **use** different forms of mathematical representation to present information correctly iii. **communicate** through lines of reasoning that are usually coherent iv. **present** work that is usually organized using a logical structure.
7–8	The student is able to: i. consistently **use** appropriate mathematical language ii. consistently **use** different forms of mathematical representation to present information correctly iii. **communicate** clearly through coherent lines of reasoning iv. present work that is consistently **organized** using a logical structure.

Criterion D: Applying mathematics in real-life contexts

Maximum: 8

At the end of year 1, students should be able to:

i. **identify** relevant elements of authentic real-life situations

ii. **select** appropriate mathematical strategies when solving authentic real-life situations

iii. **apply** the selected mathematical strategies successfully to reach a solution

iv. **explain** the degree of accuracy of a solution

v. **describe** whether a solution makes sense in the context of the authentic real-life situation.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **identify** some of the elements of the authentic real-life situation ii. **apply** mathematical strategies to find a solution to the authentic real-life situation, with limited success.
3–4	The student is able to: i. **identify** the relevant elements of the authentic real-life situation ii. **apply** mathematical strategies to reach a solution to the authentic real-life situation iii. **state**, but not always correctly, whether the solution makes sense in the context of the authentic real-life situation.
5–6	The student is able to: i. **identify** the relevant elements of the authentic real-life situation ii. **select** adequate mathematical strategies to model the authentic real-life situation iii. **apply** the selected mathematical strategies to reach a valid solution to the authentic real-life situation iv. **describe** the degree of accuracy of the solution v. **state** correctly whether the solution makes sense in the context of the authentic real-life situation.
7–8	The student is able to: i. **identify** the relevant elements of the authentic real-life situation ii. **select** adequate mathematical strategies to model the authentic real-life situation iii. **apply** the selected mathematical strategies to reach a correct solution to the authentic real-life situation iv. **explain** the degree of accuracy of the solution v. **describe** correctly whether the solution makes sense in the context of the authentic real-life situation.

Mathematics assessment criteria: Year 3

Criterion A: Knowing and understanding

Maximum: 8

At the end of year 3, students should be able to:

i. **select** appropriate mathematics when solving problems in both familiar and unfamiliar situations

ii. **apply** the selected mathematics successfully when solving problems

iii. **solve** problems correctly in a variety of contexts.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **select** appropriate mathematics when solving simple problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
3–4	The student is able to: i. **select** appropriate mathematics when solving more complex problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
5–6	The student is able to: i. **select** appropriate mathematics when solving challenging problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
7–8	The student is able to: i. **select** appropriate mathematics when solving challenging problems in both familiar and unfamiliar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.

Criterion B: Investigating patterns

Maximum: 8

At the end of year 3, students should be able to:

i. **select** and **apply** mathematical problem-solving techniques to discover complex patterns

ii. **describe** patterns as relationships and/or general rules consistent with findings

iii. **verify** and **justify** relationships and/or general rules.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **apply**, with teacher support, mathematical problem-solving techniques to discover simple patterns ii. **state** predictions consistent with patterns.
3–4	The student is able to: i. **apply** mathematical problem-solving techniques to discover simple patterns ii. **suggest** relationships and/or general rules consistent with findings.
5–6	The student is able to: i. **select** and apply mathematical problem-solving techniques to discover complex patterns ii. **describe** patterns as relationships and/or general rules consistent with findings iii. **verify** these relationships and/or general rules.
7–8	The student is able to: i. **select** and apply mathematical problem-solving techniques to discover complex patterns ii. **describe** patterns as relationships and/or general rules consistent with correct findings iii. **verify** and **justify** these relationships and/or general rules.

Note: A task that does not allow students to select a problem-solving technique is too guided and should result in students earning a maximum achievement level of 4 (year 3 and higher). However, teachers should give enough direction to ensure that all students can begin the investigation.

For year 3 and higher, a student who describes a general rule consistent with incorrect findings will be able to achieve a maximum achievement level of 6, provided that the rule is of an equivalent level of complexity.

Criterion C: Communicating

Maximum: 8

At the end of year 3, students should be able to:

i. **use** appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations

ii. **use** different forms of mathematical representation to present information

iii. **move** between different forms of mathematical representation

iv. **communicate** complete and coherent mathematical lines of reasoning

v. **organize** information using a logical structure.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **use** limited mathematical language ii. **use** limited forms of mathematical representation to present information iii. **communicate** through lines of reasoning that are difficult to interpret.
3–4	The student is able to: i. **use** some appropriate mathematical language ii. **use** different forms of mathematical representation to present information adequately iii. **communicate** through lines of reasoning that are able to be understood, although these are not always clear iv. adequately **organize** information using a logical structure.
5–6	The student is able to: i. usually **use** appropriate mathematical language ii. usually **use** different forms of mathematical representation to present information correctly iii. move between different forms of mathematical representation with some success iv. **communicate** through lines of reasoning that are clear although not always coherent or complete v. present work that is usually **organized** using a logical structure.
7–8	The student is able to: i. consistently **use** appropriate mathematical language ii. **use** different forms of mathematical representation to consistently present information correctly iii. move effectively between different forms of mathematical representation iv. **communicate** through lines of reasoning that are complete and coherent v. present work that is consistently **organized** using a logical structure.

Criterion D: Applying mathematics in real-life contexts

Maximum: 8

At the end of year 3, students should be able to:

i. **identify** relevant elements of authentic real-life situations

ii. **select** appropriate mathematical strategies when solving authentic real-life situations

iii. **apply** the selected mathematical strategies successfully to reach a solution

iv. **explain** the degree of accuracy of a solution

v. **explain** whether a solution makes sense in the context of the authentic real-life situation.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **identify** some of the elements of the authentic real-life situation ii. **apply** mathematical strategies to find a solution to the authentic real-life situation, with limited success.
3–4	The student is able to: i. **identify** the relevant elements of the authentic real-life situation ii. **select**, with some success, adequate mathematical strategies to model the authentic real-life situation iii. **apply** mathematical strategies to reach a solution to the authentic real-life situation iv. **describe** whether the solution makes sense in the context of the authentic real-life situation.
5–6	The student is able to: i. **identify** the relevant elements of the authentic real-life situation ii. **select** adequate mathematical strategies to model the authentic real-life situation iii. **apply** the selected mathematical strategies to reach a valid solution to the authentic real-life situation iv. **describe** the degree of accuracy of the solution v. **discuss** whether the solution makes sense in the context of the authentic real-life situation.

Achievement level	Level descriptor
7–8	The student is able to: i. **identify** the relevant elements of the authentic real-life situation ii. **select** appropriate mathematical strategies to model the authentic real-life situation iii. **apply** the selected mathematical strategies to reach a correct solution iv. **explain** the degree of accuracy of the solution v. **explain** whether the solution makes sense in the context of the authentic real-life situation.

Mathematics assessment criteria: Year 5

Criterion A: Knowing and understanding

Maximum: 8

At the end of year 5, students should be able to:

i. **select** appropriate mathematics when solving problems in both familiar and unfamiliar situations

ii. **apply** the selected mathematics successfully when solving problems

iii. **solve** problems correctly in a variety of contexts.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **select** appropriate mathematics when solving simple problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
3–4	The student is able to: i. **select** appropriate mathematics when solving more complex problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
5–6	The student is able to: i. **select** appropriate mathematics when solving challenging problems in familiar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.
7–8	The student is able to: i. **select** appropriate mathematics when solving challenging problems in both familiar and unfamiliar situations ii. **apply** the selected mathematics successfully when solving these problems iii. generally **solve** these problems correctly.

Criterion B: Investigating patterns

Maximum: 8

At the end of year 5, students should be able to:

i. **select** and **apply** mathematical problem-solving techniques to discover complex patterns

ii. **describe** patterns as general rules consistent with findings

iii. **prove**, or **verify** and **justify**, general rules.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **apply**, with teacher support, mathematical problem-solving techniques to discover simple patterns ii. **state** predictions consistent with patterns.
3–4	The student is able to: i. **apply** mathematical problem-solving techniques to discover simple patterns ii. **suggest** general rules consistent with findings.
5–6	The student is able to: i. **select** and apply mathematical problem-solving techniques to discover complex patterns ii. **describe** patterns as general rules consistent with findings iii. **verify** the validity of these general rules.
7–8	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to discover complex patterns ii. **describe** patterns as general rules consistent with correct findings iii. **prove**, or **verify** and **justify**, these general rules.

Note: A task that does not allow students to select a problem-solving technique is too guided and should result in students earning a maximum achievement level of 4 in year 5. However, teachers should give enough direction to ensure that all students can begin the investigation.

For year 5, a student who describes a general rule consistent with incorrect findings will be able to achieve a maximum achievement level of 6, provided that the rule is of an equivalent level of complexity.

Criterion C: Communicating

Maximum: 8

At the end of year 5, students should be able to:

i. **use** appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations

ii. **use** appropriate forms of mathematical representation to present information

iii. move between different forms of mathematical representation

iv. **communicate** complete, coherent and concise mathematical lines of reasoning

v. **organize** information using a logical structure.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **use** limited mathematical language ii. **use** limited forms of mathematical representation to present information iii. **communicate** through lines of reasoning that are difficult to interpret.
3–4	The student is able to: i. **use** some appropriate mathematical language ii. **use** appropriate forms of mathematical representation to present information adequately iii. **communicate** through lines of reasoning that are complete iv. adequately **organize** information using a logical structure.
5–6	The student is able to: i. usually **use** appropriate mathematical language ii. usually **use** appropriate forms of mathematical representation to present information correctly iii. usually move between different forms of mathematical representation iv. **communicate** through lines of reasoning that are complete and coherent v. **present** work that is usually organized using a logical structure.
7–8	The student is able to: i. consistently **use** appropriate mathematical language ii. **use** appropriate forms of mathematical representation to consistently present information correctly iii. move effectively between different forms of mathematical representation iv. **communicate** through lines of reasoning that are complete, coherent and concise v. **present** work that is consistently organized using a logical structure.

Criterion D: Applying mathematics in real-life contexts

Maximum: 8

At the end of year 5, students should be able to:

i. **identify** relevant elements of authentic real-life situations

ii. **select** appropriate mathematical strategies when solving authentic real-life situations

iii. **apply** the selected mathematical strategies successfully to reach a solution

iv. **justify** the degree of accuracy of a solution

v. **justify** whether a solution makes sense in the context of the authentic real-life situation.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **identify** some of the elements of the authentic real-life situation ii. **apply** mathematical strategies to find a solution to the authentic real-life situation, with limited success.
3–4	The student is able to: i. **identify** the relevant elements of the authentic real-life situation ii. **select**, with some success, adequate mathematical strategies to model the authentic real-life situation iii. **apply** mathematical strategies to reach a solution to the authentic real-life situation iv. **discuss** whether the solution makes sense in the context of the authentic real-life situation.
5–6	The student is able to: i. **identify** the relevant elements of the authentic real-life situation ii. **select** adequate mathematical strategies to model the authentic real-life situation iii. **apply** the selected mathematical strategies to reach a valid solution to the authentic real-life situation iv. **explain** the degree of accuracy of the solution v. **explain** whether the solution makes sense in the context of the authentic real-life situation.

Achievement level	Level descriptor
7–8	The student is able to: i. **identify** the relevant elements of the authentic real-life situation ii. **select** appropriate mathematical strategies to model the authentic real-life situation iii. **apply** the selected mathematical strategies to reach a correct solution to the authentic real-life situation iv. **justify** the degree of accuracy of the solution v. **justify** whether the solution makes sense in the context of the authentic real-life situation.

eAssessment

Students seeking **IB MYP results** for MYP mathematics courses complete an on-screen examination in which they can demonstrate their achievement of subject group objectives. Successful results can contribute to students' attainment of the **IB MYP certificate**. This verification of learning assures accurate and consistently-applied standards, as set forth in the *Guide to MYP eAssessment*.

Related concepts in mathematics

Related concept	Definition
Change	A variation in size, amount or behaviour
Equivalence	The state of being identically equal or interchangeable, applied to statements, quantities, or expressions
Generalization	A general statement made on the basis of specific examples
Justification	Valid reasons or evidence used to support a statement
Measurement	A method of determining quantity, capacity or dimension using a defined unit
Model	A depiction of a real-life event using expressions, equations or graphs
Pattern	A set of numbers or objects that follow a specific order or rule
Quantity	An amount or number
Representation	The manner in which something is presented
Simplification	The process of reducing to a less complicated form
Space	The frame of geometrical dimensions describing an entity
System	A group of interrelated elements

Mathematics glossary

Term	Definition
Authentic real-life	Relevant, meaningful and grounded in reality
Challenging	Demanding problems of high complexity that require students to have mathematical insight to be able to use knowledge and/or skills taught
Context	The setting of the problem
Familiar situations	Problems similar to those seen previously in which students are required to use knowledge and/or skills they have been taught
Form	This concept refers to the understanding that the underlying structure and shape of an entity is distinguished by its properties. Form provides opportunities for students to appreciate the aesthetic nature of the constructs used in mathematics.
Forms of mathematical representation	Words, formulae, diagrams, tables, charts, graphs and models used to represent mathematical information
Investigation	A task where, to varying degrees, students are given opportunities to pose questions, select problem-solving techniques, discover patterns, make generalizations and communicate their findings
Justification	Valid reasons or evidence that support the conclusion and explain why the rule works
Lines of reasoning	A connected sequence of steps
Logic	This concept is the basic tool used in mathematics to make conclusions about numbers, shapes and variables. Logic structures the reasoning process through which knowledge is built. It enables students to assess the truth of conclusions and transfer mathematical learning to other situations.
Logical structure	A general layout that prevents the need for going back and forth (between the task sheet and the student work and within the student work) in order to understand and follow the work
Mathematical language	The use of notation, symbols, terminology and verbal explanations
Pattern	The underlining order, regularity or predictability of the elements of a mathematical system. The repetitive features of patterns can be identified and described as relationships or general rules.
Problem-solving techniques	Strategies students use to solve problems (for example, make a table or chart, solve a simpler problem, work backwards, draw a picture, guess and check, and so on)

Term	Definition
Proof	The use of a sequence of logical steps to obtain the required result in a formal way
Relationships	This concept refers to the connections between quantities, properties or concepts; these connections may be expressed as models, rules or statements. Relationships provide opportunities for students to explore patterns in the world around them.
Teacher support	Advice given by the teacher to aid students with elements of the task (for example, to allow a student to start solving the problem)
To model	Represent
To test	Verify whether a rule works for a variety of values
Unfamiliar situations	New contexts in which students are required to use knowledge and/or skills they have been taught
Unit test	A test comprised of topics from only one branch of mathematics from the framework
Valid	A plausible solution in the context of the situation

MYP command terms for mathematics

Term	Definition
Apply	Use knowledge and understanding in response to a given situation or real circumstances. Use an idea, equation, principle, theory or law in relation to a given problem or issue.
Communicate	Express oneself in such a way that one is readily and clearly understood. Convey information about the exchange of thoughts, messages, or information through, for example, speech, signals, writing or behaviour.
Demonstrate	Prove or make clear by reasoning or evidence, illustrating with examples or practical application.
Describe	Give a detailed account or picture of a situation, event, pattern or process.
Discuss	Offer a considered and balanced review that includes a range of arguments, factors or hypotheses. Opinions or conclusions should be presented clearly and supported by appropriate evidence.
Explain	Give a detailed account including reasons or causes.
Identify	Provide an answer from a number of possibilities. Recognize and state briefly a distinguishing fact or feature.
Justify	Give valid reasons or evidence to support an answer or conclusion.
Prove	Use a sequence of logical steps to obtain the required result in a formal way.
Select	Choose from a list or group.
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
State	Give a specific name, value or other brief answer without explanation or calculation.
Suggest	Propose a solution, hypothesis or other possible answer.
Use	Apply knowledge or rules to put theory into practice.
Verify	Provide evidence that validates the result.

Selected reading

Boaler, J. 1993. "The Role of Contexts in the Mathematics Classroom: Do They Make Mathematics More 'Real'?" *For the Learning of Mathematics*. Vol 13, number 2. Pp 12–17.

Bridges, J. 1914. *The Life and Work of Roger Bacon: An Introduction to the Opus Majus (1914)*. London, UK. Williams & Norgate.

Fennell, F and Rowan, T. 2001. "Representation: An important process for teaching and learning mathematics". *Teaching Children Mathematics*. Vol 7, number 5. P 288.

Harris, K, Marcus, R, McLaren, K, and Fey, J. 2001. "Curriculum materials supporting problem-based teaching". *School Science and Mathematics*. Vol 101, number 6. Pp 310–318.

Kantrov, I. 2000. *Assessing students' mathematics learning. K–12 Mathematics Curriculum Center Issues Paper Series*. Newton, Massachusetts, USA Education Development Center, Inc.

Keiser, J. 2000. "The role of definition". *Mathematics Teaching in the Middle School*. Vol 5, number 8. Pp 506–511.

Kendal, M and Stacey, K. 2001. "The impact of teacher privileging on learning differentiation with technology". *International Journal of Computers for Mathematical Learning*. Vol 6, number 2. Pp 143-165.

Lesh, R, Post, T, and Behr, M. 1987. "Representations and Translations among Representations in Mathematics Learning and Problem Solving". *In Problems of Representation in the Teaching and Learning of Mathematics,* edited by Claude Janvier. Pp 33–40. Hillsdale, New Jersey, USA. Lawrence Erlbaum Associates.

Meyer, M, Decker, T, and Querelle, N. 2001. "Context in mathematics curricula". *Mathematics Teaching in the Middle School*. Vol 6, number 9. Pp 522–527.

Moskal, B. 2000. "Understanding student responses to open-ended tasks". *Mathematics Teaching in the Middle School*. Vol 5, number 8. Pp 500–505.

Schoenfeld, A. 2002. "Making mathematics work for all children: Issues of standards, testing, and equity". *Educational Researcher*. Vol 31, number 1. Pp 13–25.

Sullivan, P. 2011. *Teaching Mathematics: Using Research-informed Strategies*. Australian Council for Educational Research. Camberwell, Victoria, Australia. ACER Press.

Watson, A, Jones, K, and Pratt, D. 2013. *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9–19*. Oxford, UK. Oxford University Press.